

# The Modigliani-Miller Capital Structure Theorem: A “No-Arbitrage” Proof

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## 1 Introduction

In finance, we typically assume that the objective of the corporation is to maximize the value of the firm’s equity. This follows as a logical corollary of the famous Fisher Separation Theorem (cf. Fisher (1930)), which notes the following: (i) the firm’s investment decision is independent of shareholder (risk bearing and consumption) preferences; and (ii) the investment decision is independent of the financing decision. Since the capital markets enable shareholders to satisfy their own unique consumption and risk preferences by trading securities, by maximizing firm value management indirectly enables the firm’s shareholders to maximize their own expected utilities.

More than fifty years ago, Modigliani and Miller (1958, 1961) published a pair of articles which provide the theoretical foundations for the so-called “modern” theory of corporate finance. In these articles, they propose various theorems concerning the financial policies of publicly traded corporations. The Modigliani and Miller theorems are based upon the very important concept of arbitrage. Essentially, they show that in a world without economic “frictions” or “imperfections” such as taxes, transaction costs, bankruptcy costs, and agency costs, then the corporate financial policies (including decisions such as how much debt to issue and how much to pay out in dividends) cannot possibly affect corporate value. While these assumptions may make finance seem more like physics than a social science, this line of reasoning also makes it possible to develop more sensible and robust theories concerning the conditions under which corporate financial policies *do* matter. Corporate financial policies take place in the context of actively traded, informationally efficient financial markets in which investors are poised to take advantage of any pricing disparities which may appear. If a particular asset becomes overpriced, then one way to make profits is to short sell this asset and use the proceeds from the short sale to fund investment in an otherwise identical security which is not overpriced. The resulting “hedge” portfolio involves zero risk (because the risks of the two positions perfectly offset each other), zero net investment, and positive returns. In other words, this is the financial equivalent of a “free lunch”.

## 2 Capital Structure Policy and the Value of the Firm

Perhaps the best known Modigliani and Miller theorem is their capital structure theorem. Modigliani and Miller show that in a “frictionless” economy (i.e., one in which there are no

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taxes, transaction costs, bankruptcy costs, or agency costs), then the market value of the firm is independent of its capital structure. To see this, suppose there is a firm (let's call it MM, Inc.) that is entirely equity financed and expects to have operating income of \$1,500 in perpetuity.<sup>2</sup> MM has a market value of  $V = \$10,000$ , which is accounted for by 1,000 outstanding shares with a price per share of \$10. We assume that there exist three possible states of nature in which MM produces the following state-contingent operating incomes, earnings per share, and returns:

<b>TABLE 1 (unlevered shares)</b>				
<b>OUTCOMES</b>	<b>STATE 1</b>	<b>STATE 2</b>	<b>STATE 3</b>	<b>MEAN</b>
Operating Income	\$500	\$1,000	\$2,000	\$1,500
EPS	\$0.50	\$1.00	\$2.00	\$1.50
$r$	5%	10%	20%	15%

Should MM lever itself, it will perform a debt for equity swap in which it issues \$5,000 in bonds at par with a coupon rate of 10% and repurchases 500 shares of stock @ \$10 per share. Such an operation will reduce outstanding shares to 500 with a total market value of \$5,000. The resulting state-contingent operating incomes, earnings per share, and returns are summarized in Table 2:

<b>TABLE 2 (levered shares)</b>				
<b>OUTCOMES</b>	<b>STATE 1</b>	<b>STATE 2</b>	<b>STATE 3</b>	<b>MEAN</b>
Operating Income	\$500	\$1,000	\$2,000	\$1,500
Interest	\$500	\$500	\$500	\$500
EPS	\$0.00	\$1.00	\$3.00	\$2.00
$r$	0%	10%	30%	20%

Next, we consider a scenario in which a hypothetical investor levers the unlevered firm by buying stock on margin. By purchasing 4 shares of stock for \$40 and borrowing \$20 for a net investment of \$20, this investor will be able to transform the unlevered state-contingent returns summarized in Table 1 into the levered state-contingent returns summarized in Table 2:

<b>TABLE 3 (transformation of unlevered into levered shares)</b>				
<b>OUTCOMES</b>	<b>STATE 1</b>	<b>STATE 2</b>	<b>STATE 3</b>	<b>MEAN</b>
Operating Income	\$500	\$1,000	\$2,000	\$1,500
Earnings on 4 shares	\$2.00	\$4.00	\$8.00	\$6.00
Margin interest	\$2.00	\$2.00	\$2.00	\$2.00
$r$	0%	10%	30%	20%

<sup>2</sup> This numerical example is actually taken from the 17<sup>th</sup> chapter of Brealey and Myers (2002). See their "Macbeth Spot Removers" example.

By inspection, we see that the investor has been able to lever the unlevered firm, thereby producing (on personal account) the same set of state-contingent returns that the firm would offer if it were levered on corporate account.

Next, we consider an alternative scenario in which the hypothetical investor unlevers the levered firm by purchasing 1 share of stock for \$10 and lending \$10 for a net investment of \$20. In essence, the investor has combined a short position in corporate leverage with a long position in personal leverage, thereby producing an unlevered share. As a result, the investor will be able to transform the levered state-contingent returns summarized in Table 2 into the unlevered state-contingent returns summarized in Table 1:

<b>OUTCOMES</b>	<b>STATE 1</b>	<b>STATE 2</b>	<b>STATE 3</b>	<b>MEAN</b>
Operating Income	\$500	\$1,000	\$2,000	\$1,500
Earnings on 1 share	\$0.00	\$1.00	\$3.00	\$2.00
Earned interest	\$1.00	\$1.00	\$1.00	\$1.00
Net earnings	\$1.00	\$2.00	\$4.00	\$3.00
$r$	5%	10%	20%	15%

Since the investor is able to replicate on personal account the state-contingent returns offered by the firm regardless of whether it levers itself or remains totally equity financed, she will not be willing to pay a premium price for the shares of a firm due to the financing activities it undertakes, since there is no value added.

Suppose that MM is all equity financed, and an otherwise identical levered firm (which we will call NN) exists whose shares trade at a premium relative to MM's shares. Then there is an arbitrage opportunity. Investors can create a zero net investment, zero risk, positive return hedge portfolio by leveraging MM on personal account with borrowed money and proceeds from short selling NN. Similarly, if MM traded at a premium relative to NN, then investors can create a zero net investment, zero risk, positive return hedge portfolio by purchasing a portfolio consisting of NN and a riskless bond with proceeds obtained from short selling MM.

### 3 References

Brealey, Richard C. and Stewart C. Myers, 2002, *Principles of Corporate Finance*, 7<sup>th</sup> edition, New York: McGraw Hill (see pp. 93-97 at the following link: <http://bit.ly/KUNbQ4>).

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